penetrator mass and diameter; sea-ice thickness, tensile strength, shear strength, and elastic modulus; and an empirical factor related to plug geometry. Moreover, it was possible to relate the sea-ice mechanical strength properties to temperature and salinity content through the Assur theory governing sea-ice strength. Theoretical values of critical impact velocity were determined from characteristic experimental data. Agreement between theory and experiment was good.

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A Study of Prediction Techniques for Aircraft Carrier Motions at Sea

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Mathematical techniques for calculating ship motion time histories are developed for application to aircraft carrier landing operations. Various methods for short-term prediction of motion time history are considered, based on both deterministic and statistical techniques. The most attractive is the deterministic technique based on a convolution integral representation, with wave height measurements at the bow serving as input. A kernel-type weighting function, operating on the input to provide the predicted motion history, is derived from ship response functions. It yields pitch prediction up to 6 sec ahead on the basis of model test data. The limitations of classical statistical prediction techniques, as well as practical implementation difficulties, are shown. Recent prediction theory developments (i.e., Kalman filtering) are used in a "hybrid" prediction technique (containing elements of both deterministic and statistical approaches) that is expected to increase the prediction time.

Nomenclature

A'_{33} = vertical sectional added mass
a = amplitude of surface wave elevation
a_{ij} = matrix element for vertical plane motions
B^* = local beam of ship section
$F(\omega_e;V,\beta)$ = frequency-dependent part of spatial transport delay
g = acceleration due to gravity
$K_z(t),K_\theta(t)$ = kernel functions for heave and pitch, respectively
$M_w(t)$ = pitch moment due to waves, positive about the y axis
S = cross-sectional area of ship
T = prediction time
$T_{z\eta},T_{\theta\eta}$ = response operator (amplitude and phase relative to wave) for heave and pitch, respectively

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t = time
$u(t)$ = unit step function
V = forward speed
w_0 = vertical wave orbital velocity
x,y,z = orthogonal axis system; x axis horizontal, positive toward the bow; y axis horizontal, positive to port; z axis vertical, positive upward. Symbol (z) also used to represent heave motion relative to an equilibrium position.
x_b,x_s = bow and stern x coordinates, respectively
x_1 = x-coordinate location of wave measurement point
$Z_w(t)$ = vertical wave force, positive upward
β = heading angle of waves relative to ship
η_m = wave elevation measured at a moving point
θ = pitch angle, positive for rotation about y axis
λ = wavelength
ρ = fluid mass density
$\sigma_\epsilon,\sigma_\theta$ = rms values of prediction error and pitch angle, respectively
τ_e = effective time extent of kernel function
ϕ_z,ϕ_θ = phase angles of heave and pitch, respectively, with regard to wave reference
ω = circular frequency (rad/sec)
ω_e = circular frequency of encounter
$|\ |$ = absolute value

Introduction

MOTIONS due to waves acting on an aircraft carrier at sea influence the landing of aircraft, since varying deck displacements due to the ship motions affect the pilot's ability to carry out a safe landing.

Analytical studies were previously carried out at Oceanics, Inc. to provide information on aircraft carrier vertical plane motion characteristics (i.e., heave and pitch) for different idealized sea states.[1] This information, in the form of frequency response curves and spectral densities, was developed as a means of representing ship motion characteristics for utilization in a systems analysis of the entire aircraft carrier landing process.[2] The information developed in Ref. 1 could be used to develop an analog shaping filter which, in conjunction with a random white noise generator, would produce instantaneous motion time histories as a simulation tool. However, the time histories would only be representative of the ship motions up to the "present" time instant, which is the observation time of the motion occurrence.

In the system analysis study of the aircraft carrier landing process using an optical landing system,[2] a significant improvement in the entire operation (by virtue of reduced accident rate, waveoffs, etc.) was shown to exist if vertical plane ship motion time histories were predicted successfully and the information incorporated into the landing procedure. A separate analysis[3] has shown that the minimum prediction time for implementation, which accounts for the time needed by the aircraft to perform appropriate maneuvers after receiving the command, is 5 sec, and prediction times greater than that amount would enhance this technique even further. Such predictions of ship motion also would be significant for an automatic carrier landing system, since the information transmitted to the aircraft is terminated some 5 sec prior to touchdown.

As a result of the significance of ship motion prediction as an aid in the landing process, a program was carried out to obtain an analytical development of the required predictor, together with a means for verifying the results by comparison with model studies in a wave tank. The techniques used for this program, and the results of the study, are described in the present paper. Throughout this paper, which is concerned with the concept of prediction, it must be understood that the term "prediction" implies short-term prediction of the instantaneous time histories of the particular modes of motion, prior to their occurrence. The prediction is a continuous time record that can be easily compared with the actual record of the quantity itself, in order to assess the degree of prediction capability.

Techniques Used in Analysis

While most ship motion studies are represented in the form of spectra for particular sea state conditions, limited studies have been devoted to the problem of presenting time histories of the motion for a particular sea condition.[4,5] These studies have been primarily concerned with duplication and presentation of the actual motions experienced by a ship model in an irregular wave system generated in a wave tank. The time domain representation of the motions has been based upon convolution integral operations on the wave histories obtained at some point located ahead of the bow of the ship. The weighting function operating on the waves has been shown to be the impulse response function of the ship response to a wave input, with mathematical derivations of this form based upon the assumption that the dynamic system represented by a ship in waves is the same as a simple mechanical (or electrical) dynamic system subjected to an arbitrary forcing function. The derivations in these cases do not adequately account for the fact that the hydrodynamic system (a ship) experiences forces due to waves that result from a spatial distribution of the waves, as well as their time variation. This is due to the fact that when a wave impulse (with respect to time) occurs at a fixed point, there is an associated wave system present throughout all space, and the ship will experience a force due to this wave system associated with the localized wave impulse. In view of this spatial distribution of wave effects, and its influence in determining forces on a ship located some distance from a reference point, an alternate derivation of the time domain representation of ship motions is outlined herein. The form of the resulting time domain operator, and its relation to those derived by previous investigators, are discussed. The operations necessary for transforming a mathematical function that is used for motion reproduction (i.e., time history up to the present observation time) into one that functions as a predictor is also presented.

The concept of carrying out predictions, when using as input the measurements of the waves at a point located ahead of the ship bow, is based upon the fact that the forcing elements are the waves, and that when these waves are measured in advance of their action on the ship an appropriate prediction of the motions can be obtained. Since the waves that excite significant heave and pitch motions of an aircraft carrier are approximately 0.75–1.0 times the ship length, a measurement of the waves at a point just ahead of the bow provides a significant phase lead useful for a prediction system. This heuristic argument must be verified for the present case of an aircraft carrier, since previous studies applied to other, shorter ships have only been able to provide motion reproduction, with one exception[6] where prediction was obtained for a wave measuring point located at a distance equal to half the ship length ahead of the bow. The requirements as to the location of the wave measuring point and the significance of ship size and speed on the prediction capability using this approach are considered in this paper.

In contrast to this technique of prediction based upon measurement of the waves ahead of the ship, other prediction techniques are also considered. The most appropriate one is the Wiener[7] prediction method which is a statistical technique, where the predictor is derived on the basis of knowledge of the spectral characteristics of the stochastic variable under consideration. The technique is generally known and outlined in various books concerned with random processes.[8] The technique based upon a convolution operation on the waves measured at the ship bow is a deterministic method of reproducing the motion time history, and even when extended as a predictor the technique remains deterministic, i.e., it is independent of the statistical characteristics of the ship motion or the waves. An outline of each of these techniques is given in this paper, and inherent advantages and disadvantages are discussed. Certain "hybrid" techniques, which incorporate elements of each procedure, are also considered, and the relative advantages of that approach are portrayed as a conceptual method for extending prediction time.

Analysis of Impulsive Waves and Resulting Hydrodynamic Forces on a Ship

To carry out an analysis of the properties of a wave system relative to a moving ship, a coordinate system is chosen with its origin on the undisturbed water surface at the ship LCG position. This coordinate system has the x axis positive toward the bow (in the direction of forward motion), the y axis positive to port, and the z axis positive upward. Waves on the free surface are functions of time t, and the two space variables x and y. For the present analysis, all of the waves are assumed to be traveling in the same direction (i.e., unidirectional seas), and they are observed at a point along the x axis (i.e., $y = 0$). To account for wave motion relative to a moving ship, the frequency domain is that of the (circular) encounter frequency ω_e, which is defined by

$$\omega_e = \omega + (\omega^2/g)V\cos\beta \tag{1}$$

Equation (1) is assumed to represent waves in the range of head to beam seas, which are the predominant wave conditions expected to be encountered during aircraft carrier landing operations.

A wave disturbance measured at a point x_1 ahead of the translating ship bow is expressed in Fourier form as

$$\eta_m(x_1,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} N_1(x_1,\omega_e)e^{i\omega_e t}d\omega_e \tag{2}$$

This same wave at a point x, which is located downstream along the x axis (not necessarily along the direction of wave travel from x_1), is observed with a phase lag of $(2\pi/\lambda)(x_1 - x)\cos\beta$, where λ is the wavelength of the wave given by

$$\lambda = 2\pi g/\omega^2 \tag{3}$$

This phase lag, or spatial transport delay, is derived on the basis of linear wave theory, and follows as a generalization of similar equations presented by Davis and Zarnick.[9] The phase lag is represented as a function of ω_e by

$$\frac{2\pi}{\lambda}(x_1 - x)\cos\beta = \frac{\omega^2}{g}(x_1 - x)\cos\beta = \frac{(\omega_e - \omega)}{V}(x_1 - x) = F(\omega_e; V,\beta)(x_1 - x) \tag{4}$$

since ω is a function of ω_e in accordance with Eq. (1). The resulting expression for the wave disturbance at the point x is given by

$$\eta_m(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} N_1(x_1,\omega_e)e^{i\omega_e t - iF(\omega_e)(x_1-x)}d\omega_e \tag{5}$$

which by Fourier inversion is represented in the form

$$\eta_m(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\eta_m(x_1,\tau)d\tau \times \int_{-\infty}^{\infty} e^{i\omega_e(t-\tau) - iF(\omega_e)(x_1-x)}d\omega_e \tag{6}$$

With the wave disturbance known as a function of time and position along the x axis in terms of a measured value at the point x_1 (for all values of $x < x_1$), the hydrodynamic forces acting on a ship placed within this wave field are found by application of a slender-body theory (known in ship hydrodynamics as strip theory). As an example, the hydrostatic vertical wave force at a local ship section is given by

$$dZ_w^{(1)}/dx = \rho g B^*\eta_m(x,t) \tag{7}$$

where B^* is the local section beam. The total hydrostatic wave force is then found by integrating over the length of the ship that is affected by this wave disturbance, leading to a force given by

$$Z_w^{(1)} = \frac{\rho g}{2\pi}\int_{-\infty}^{\infty}\eta_m(x_1,\tau)\int_{-\infty}^{\infty} e^{i\omega_e(t-\tau) - iF(\omega_e)x_1} \times \left[\int_{x_s}^{x_b} B^* e^{iF(\omega_e)x}dx\right]d\omega_e d\tau \tag{8}$$

Similarly, the hydrostatic contribution to the pitch moment due to this wave system is

$$M_w^{(1)} = -\frac{\rho g}{2\pi}\int_{-\infty}^{\infty}\eta_m(x_1,\tau)\int_{-\infty}^{\infty} e^{i\omega_e(t-\tau) - iF(\omega_e)x_1} \times \left[\int_{x_s}^{x_b} xB^* e^{iF(\omega_e)}dx\right]d\omega_e d\tau \tag{9}$$

Considering the additional terms that enter into the total wave force due to the hydrodynamic inertial effects, simple interpretations of these force components in terms of operations on the waves are easily developed. The hydrodynamic part of the wave force, obtained by application of slender-body theory[10,11] extended to the surface ship case, results in a local force on a ship section given by

$$dZ_w^{(2)}/dx = \rho S Dw_0/Dt + D/Dt(A'_{33}w_0) \tag{10}$$

where the operator

$$D/Dt = \partial/\partial t - V\partial/\partial x \tag{11}$$

Equation (10) is rewritten in the form

$$dZ_w^{(2)}/dx = (\rho S + A'_{33})(Dw_0/Dt) + VA'_{33}(\partial w_0/\partial x) \tag{12}$$

after recognizing the resulting terms following integration over the ship length to obtain the total hydrodynamic wave force. The vertical orbital velocity w_0 is

$$w_0(x,t) = D\eta_m(x,t)/Dt \tag{13}$$

so that

$$\frac{Dw_0}{Dt} = \frac{D^2\eta_m}{Dt} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\eta_m(x_1,\tau)d\tau \times \int_{-\infty}^{\infty}\omega^2(\omega_e)e^{i\omega_e(t-\tau) - iF(\omega_e)(x_1-x)}d\omega_e \tag{14}$$

and

$$\frac{\partial w_0}{\partial x} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\eta_m(x_1,\tau)d\tau \times \int_{-\infty}^{\infty}\omega(\omega_e)F(\omega_e)_e{}^{i\omega_e(t-\tau) - iF(\omega_e)(x_1-x)}d\omega_e \tag{15}$$

Inserting into Eq. (12) and integrating over the ship leads to

$$Z_w^{(2)} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\eta_m(x,\tau)\int_{-\infty}^{\infty} e^{i\omega_e(t-\tau) - iF(\omega_e)x_1} \times \left[\omega^2(\omega_e)\int_{x_s}^{x_b}(\rho S + A'_{33})e^{iF(\omega_e)x}dx + V\omega(\omega_e)F(\omega_e)\int_{x_s}^{x_b}A'_{33}e^{iF(\omega_e)x}dx\right]d\omega_e d\tau \tag{16}$$

and the total wave force is given by the sum of Eqs. (8) and (16), viz.,

$$Z_w = Z_w^{(1)} + Z_w^{(2)} \tag{17}$$

Following the previous derivations, it can be shown that

$$\frac{dM^{(2)}}{dx} = -(\rho S + A'_{33})x\frac{Dw_0}{Dt} - VA'_{33}\left(w_0 + x\frac{\partial w_0}{\partial x}\right) \tag{18}$$

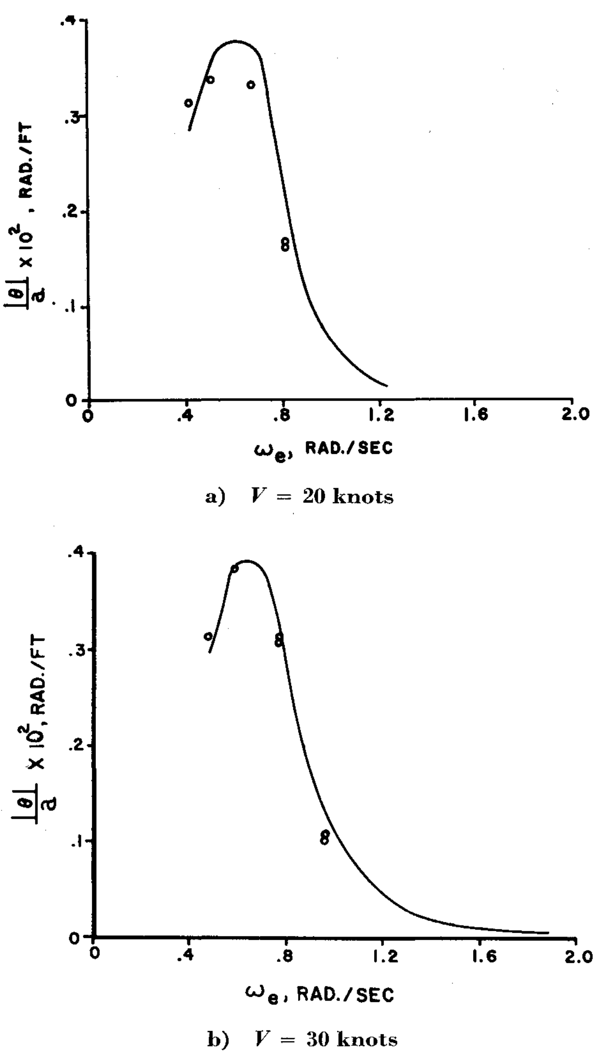


Fig. 1 Comparison of full-scale theoretical and experimental pitch amplitude response in head seas; — theory, ∘ MIT test data.

so that the total hydrodynamic wave-induced pitch moment is

$$M^{(2)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \eta_m(x_1,\tau) \int_{-\infty}^{\infty} e^{i\omega_e(t-\tau)-iF(\omega_e)x_1} \times \left[\omega^2(\omega_e)\int_{x_s}^{x_b} x(\rho S + A'_{33})e^{iF(\omega_e)x}\,dx + V\omega(\omega_e)F(\omega_e) \times \int_{x_s}^{x_b} xA'_{33}e^{iF(\omega_e)x}dx - iV\omega(\omega_e)\int_{x_s}^{x_b} A'_{33}e^{iF(\omega_e)x}dx\right] d\omega_e d\tau \tag{19}$$

and the total pitch moment due to the arbitrary wave disturbance is given by

$$M_w = M_w^{(1)} + M_w^{(2)} \tag{20}$$

In all the expressions in Eqs. (1–20), sign ω^2/g = signω and signω_e = signω (for $V\cos\beta \geq 0$), so that the Fourier integral operations represent real functions. The x integrations result in complex functions of ω_e, and it can be shown that the real parts are even functions of ω_e, and the imaginary parts odd functions of ω_e.

As a result of the previous analysis, the vertical force and pitch moment acting on a translating ship are given in terms of mathematical operations on the wave time history measured at a moving reference point (x_1) that is located at a fixed distance ahead of the ship bow. This result exhibits the precise form (based on strip theory) of the force and moment associated with an arbitrary wave disturbance that is measured prior to its "contact" with the ship, and illustrates the effects of the spatial distribution of free surface waves under the assumption of long-crested unidirectional waves producing head (or bow) seas.

Ship Motion Response to Arbitrary Unidirectional Waves

The heave and pitch motions of a ship in waves are given by the solution of the coupled equations of motion

$$a_{11}\ddot{z} + a_{12}\dot{z} + a_{13}z + a_{14}\ddot{\theta} + a_{15}\dot{\theta} + a_{16}\theta = Z_w \tag{21}$$

$$a_{21}\ddot{z} + a_{22}\dot{z} + a_{23}z + a_{24}\ddot{\theta} + a_{25}\dot{\theta} + a_{26}\theta = M_w \tag{22}$$

where the values of the a_{ij} are given in Ref. 1. With the wave force and moment given by the previous equations, the solution for the motions can be shown to be

$$z = \frac{1}{2\pi}\int_{-\infty}^{\infty} \eta_m(x_1,\tau)d\tau \int_{-\infty}^{\infty} e^{i\omega_e(t-\tau)-iF(\omega_e)x_1}T_{z\eta}(\omega_e)d\omega_e \tag{23}$$

$$\theta = \frac{1}{2\pi}\int_{-\infty}^{\infty} \eta_m(x_1,\tau)d\tau \int_{-\infty}^{\infty} e^{i\omega_e(t-\tau)-iF(\omega_e)x_1}T_{\theta\eta}(\omega_e)d\omega_e \tag{24}$$

where

$$T_{z\eta}(\omega_e) = |z/a|e^{i\phi_z} \tag{25}$$

and

$$T_{\theta\eta}(\omega_e) = |\theta/a|e^{i\phi_\theta} \tag{26}$$

are the complex "transfer functions" of the heave and pitch motions of a ship with respect to sinusoidal waves referred to the CG position.

The transport lag $e^{-iF(\omega_e)x_1}$ is combined with the standard transfer function referred to the CG to form the transfer function of the ship motion relative to sinusoidal waves at the point $x = x_1$, i.e.,

$$T_{z\eta}(\omega_e)e^{-iF(\omega_e)x_1} = T_{z\eta}(\omega_e;x_1) \tag{27}$$

$$T_{\theta\eta}(\omega_e)e^{-iF(\omega_e)x_1} = T_{\theta\eta}(\omega_e;x_1) \tag{28}$$

so that the ship motion representations are simplified to the forms

$$z(t) = \int_{-\infty}^{\infty} K_z(t-\tau)\eta_m(\tau)d\tau \tag{29}$$

$$\theta(t) = \int_{-\infty}^{\infty} K_\theta(t-\tau)\eta_m(\tau)d\tau \tag{30}$$

where

$$K_z(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} T_{z\eta}(\omega_e;x_1)e^{i\omega_e t}d\omega_e \tag{31}$$

$$K_\theta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} T_{\theta\eta}(\omega_e;x_1)e^{i\omega_e t}d\omega_e \tag{32}$$

and it is understood that $\eta_m(\tau)$, $K_z(t)$, $K_\theta(t)$ are specifically associated with measurements made at and/or referred to the point $x = x_1$. The kernel functions $K_z(t)$ and $K_\theta(t)$ are defined in Eqs. (31) and (32) as inverse Fourier transforms of the ship motion transfer functions $T_{z\eta}$ and $T_{\theta\eta}$ (relative to the wave motion at the point $x = x_1$), and as such they are considered as the impulse response functions for the ship heave and pitch motions, analogous to the same function for mechanical or electrical dynamic systems.[8] The basic representations of ship motion given by Eqs. (29–32) are essentially the same as those derived in previously cited references,[5–7] but they have greater applicability and are more precise in certain respects. In particular, the results are applicable to oblique unidirectional irregular waves (ranging from head to beam seas); the precise form of the spatial dependence of the wave vertical force and moment is presented; special distinction is made between the basic wave frequency and the frequency of encounter with the waves; and the use of constant coefficient differential equations for the present case of an aircraft carrier avoids certain difficulties inherent in applying frequency-dependent coefficient equation systems to arbitrary motion analyses. These facts allow an orderly development of the fundamental relations, but they are still limited by the defects inherent in the basic theoretical model, viz., strip theory. The final validity of the technique must therefore be demonstrated by application and by comparison with experimental data.

For application to the aircraft carrier, the only data initially available for evaluation of the impulse response functions were theoretical results.[1] Since the final measure of the effectiveness of motion time history reproduction is obtained by comparison of the results of Eqs. (28–32) (with wave motion time history at a point ahead of the bow as the input) with experimental ship motions, it is necessary to insure that the impulse response functions are valid for the test conditions. An experimental program was conducted at the Massachusetts Institute of Technology Ship Model Towing Tank to obtain ship response characteristics in regular head sea waves, as well as ship motion time history data (and wave input data) for irregular wave conditions. The tests were carried out in head seas over a range of full scale speeds from 10 to 30 knots. The regular wave tests included wavelengths in the range $0.50 \leq \lambda/L \leq 2.0$, whereas the irregular wave tests were conducted in wave systems that have the same significant wave heights as Sea State 5 and 6 and encompass a frequency range sufficient to excite significant ship motions. The ship model was that of the USS Forrestal, constructed at a scale ratio of $\frac{1}{144}$, and the waves were measured by a sonic transducer placed at a point equivalent to 50 ft (full scale) ahead of the forward perpendicular.

A comparison of theoretical[1] and experimental amplitude responses for pitch motion, for the forward speeds of 20 and 30 knots, is shown in Fig. 1. The theory for pitch has good agreement with the experiments, whereas that for heave (not shown) exhibits some departures in portions of the frequency band of interest. New model test data[13] will provide the necessary information for heave response characteristics. In view of the good agreement in the case of pitch, it is expected that the impulse response function $K_\theta(t)$ obtained from the theoretical response characteristics[1] will provide good duplication of pitch motion time history, and that similar agreement will be obtained for heave motion when the new experimental data are applied.

It can be shown that the amplitude responses of the heave and pitch are even functions of the frequency of encounter ω_e, whereas phase angles are odd functions, so that the impulse functions are

$$K_z(t) = \frac{1}{\pi}\int_0^\infty \left|\frac{z}{a}\right| \cos[\omega_e t + \phi_z - F(\omega_e)x_1]d\omega_e \quad (33)$$

$$K_\theta(t) = \frac{1}{\pi}\int_0^\infty \left|\frac{\theta}{a}\right| \cos[\omega_e t + \phi_\theta - F(\omega_e)x_1]d\omega_e \quad (34)$$

Numerical computations of pitch impulse functions for the case of head seas were carried out and representative results are shown in Fig. 2. The impulse (kernel) functions have values for both positive and negative values of their argument, which thereby differs from results obtained for the impulse response function derived for linear dynamic systems.[8] In the case of a dynamic system, the input is a force that produces the response motion, and the system has no response prior to the application of the input. With the response represented in the form of a convolution-type integral similar to that shown in Eqs. (29) and (30) (with the force input in the place of η_m), the impulse response function must be identically zero for negative values of its argument since the "effect" (system motion) cannot precede the "cause" (force input). This requirement of zero values of the impulse response function for negative time is known as the condition of physical realizability, since values for negative values of the kernel argument would lead to the incorrect conclusion that response at the present instant requires knowledge of future values of the input. The kernel functions in the present case of a ship with a wave record as the input are not physically realizable in this sense since the wave does not cause the ship motion, but the force and the moment associated with the wave are the causative inputs. This interpretation has been presented and discussed in other studies[5,9] where the hydrodynamic relations between wave and force, as well as the effect of the spatial variation of gravity waves, have also been considered in arriving at this explanation.

A particular application of the convolution integral representation of pitch motion time history, using the entire range of values $K_\theta(t)$ (including the values for negative arguments), was carried out for a Forrestal class carrier at conditions equivalent to a 20-knot forward speed in Sea State 6 head waves. Good reproduction of the pitch motion was found for this case, but the motion was only determined to "present" time by operating on wave data about 5 sec (full scale) in the "future" of the required motion observation time. This is due to the nature of the kernel function that is not physically realizable. Since the magnitudes of the pitch kernel are small in the domain of negative time, the kernel was modified by neglecting the part for negative time and thereby obtaining a physically realizable kernel function that is only defined for positive values of its argument. Computation of the pitch motion time history for the same conditions, using this new kernel

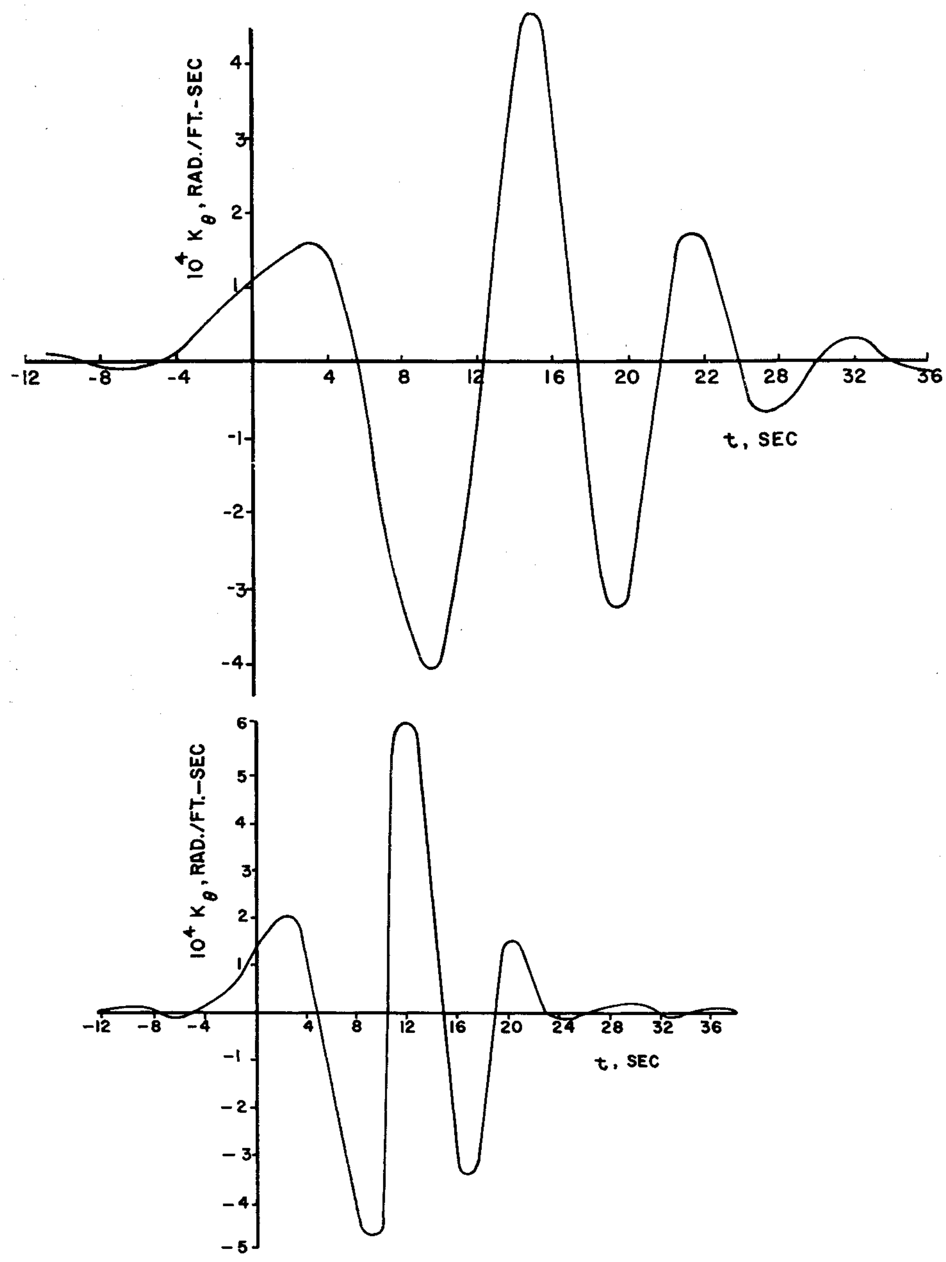
$$K_\theta(t) = \begin{cases} K_\theta(t), & t > 0 \\ 0\quad, & t < 0 \end{cases} \quad (35)$$

resulted in motion reproduction that was also in good agreement with the experiments, as shown in Fig. 3. The numerical values were also quite close to those obtained for the complete kernel (including values for negative time), and hence it appears that adequate motion time history reproduction can be obtained with a physically realizable kernel. An important aspect of this result is that only past and present wave input data are required to obtain motion data up to the present time instant.

To simplify and limit the discussion of the many aspects to be considered in a practical program of this nature, no further consideration will be given to heave motions in the remaining portion of this paper. Since the carrier stern ramp excursions are predominantly due to pitch motions, consideration will be devoted to that particular mode of motion at this time in order to judge the applicability of the techniques developed herein.

Development of a Pitch Predictor Kernel Function

The results obtained have indicated that adequate pitch time history reproduction can be obtained by operating on the wave time history measured at a point ahead of the ship bow in random head seas. This information is obtained for the present instant of time by operating on the present and past history of the wave input data, by use of a physically realizable kernel obtained by neglecting small values of the kernel magnitude for negative values of its argument. Following this approach, it appears plausible that neglecting part of the pitch kernel function for a small segment of positive time from $t = 0$ up to some value $t = T$ [i.e., by replacing the kernel function $K_\theta(t)$ by $K_\theta(t)\cdot u(t - T)$, where $u(t - T)$ is the unit step function] can provide some sort of prediction of the motion up to T sec ahead of the present time. This



a) $V = 20$ knots

b) $V = 30$ knots

Fig. 2 Full-scale pitch kernel function for head seas.

neglect of the kernel function for a portion of positive time will be referred to as a "truncated" kernel, where the truncation is performed on the left-hand (i.e., small-time) end of the curve.

The capability of prediction by this approach can be demonstrated mathematically by expressing Eq. (30) in an alternate form, viz.,

$$\theta(t) = \int_{-\infty}^{t} K(t - \tau)\eta_m(\tau)d\tau \tag{36}$$

since $K(\tau)$ is physically realizable [see Eq. (35)]. For practical consideration it can be seen that the pitch kernel

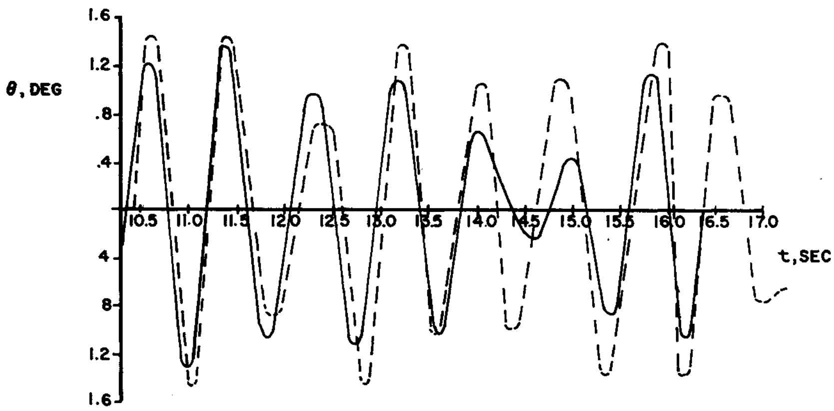
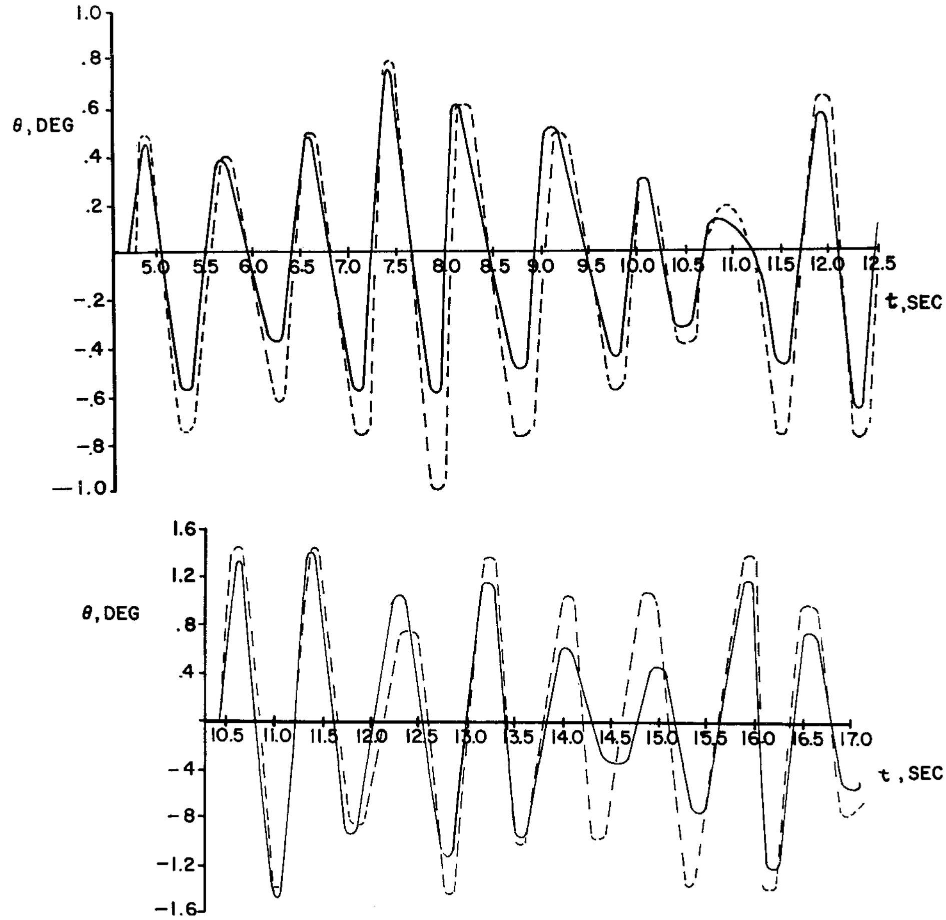


Fig. 3 Comparison of theoretical pitch motion reproduction with experimental model data for head seas, $V = 20$ knots; — theory, - - MIT experimental data.



a) **Prediction time $T = 0.36$ sec**

b) **Prediction time $T = 0.48$ sec**

Fig. 4 Comparison of predicted pitch motion with experimental model data for head seas, V = 20 knots; — predicted, – – MIT experimental data.

functions effectively terminate at some finite value of time τ_e, which is of the order of 36 sec (full scale) for a Forrestal class aircraft carrier (see Fig. 2). Thus, the convolution operation of Eq. (36) requires a proper weighting of the past history of the input $\eta_m(\tau)$ for a time extent of τ_e sec, which is expressed by

$$\theta(t) = \int_{t-\tau_e}^{t} K(t - \tau)\eta_m(\tau)d\tau \qquad (37)$$

This last expression yields the value $\theta(t)$ at present time t, with input data at time t and for the past τ_e sec. If the kernel function is truncated, as previously described, it is only necessary to operate on $(\tau_e - T)$ sec of the past history of $\eta_m(\tau)$ to obtain a good approximation to $\theta(t)$. However, the maximum allowable information on the past history of $\eta_m(\tau)$ is available, i.e., past history up to τ_e sec in the past, so that operation on the additional T sec of wave input available will yield a value of $\theta(t + T)$ that is a deterministic prediction for T sec ahead of present time. This may be expressed mathematically after recognizing that the truncated kernel can be represented by

$$K(t)u(t - T) = \begin{cases} K(t + T), & t > 0 \\ 0 & , \ t < 0 \end{cases} \qquad (38)$$

which leads to

$$\theta(t + T) = \int_{t-\tau_e}^{t} K(t + T - \tau)\eta_m(\tau)d\tau \qquad (39)$$

To check the prediction capability of the truncated kernel method just described, comparisons of measured and predicted pitch motions were made using experimental data.[12] The results of this comparison are shown in Figs. 4–6 which are appropriate to head seas at speeds of 20 and 30 knots, for different simulated sea conditions. Since these data are obtained from model tests, the time scale is reduced by a factor of 12, and hence the actual full-scale prediction times are obtained by multiplying the scaled prediction time by this factor. It can be seen from these figures that a prediction time of just under 6 sec, with adequate prediction capability, is obtained by application of this procedure. This prediction time is slightly more than the minimum required useful prediction time,[3] and the analysis of the rms prediction error for these three records results in a relative error in the range from 0.36 to 0.49, where the relative error is defined to be the rms error divided by the rms amplitude of the actually measured variable, viz., $\sigma_\epsilon/\sigma_\theta$. The values obtained for the pitch prediction in this study thus indicate an appreciable gain possible in the safety of aircraft landing operations.[3]

Since a good prediction of the pitch motion has been obtained by using the truncated kernel function, it is important to determine the basis for this good performance. The significant point discussed previously is the fact that good motion time history reproduction could be obtained with a truncated kernel function $[K_\theta(t) = 0, 0 \leq t \leq T]$ using T sec less of wave input data, and hence it is important to see what characteristics of the ship response are responsible for this result. The method used for analysis is examination of the frequency response characteristics of the ship motion, which can be obtained by applying a Fourier transform to Eq. (30), resulting in

$$\bar{\theta}(\omega_e) = \bar{K}_\theta(\omega_e)\bar{\eta}_m(\omega_e) \qquad (40)$$

where the bar symbols over the variables represent the Fourier transforms. Assuming the pitch kernel is physically realizable, it is represented in the form

$$K_\theta(t) = G(t)u(t) \qquad (41)$$

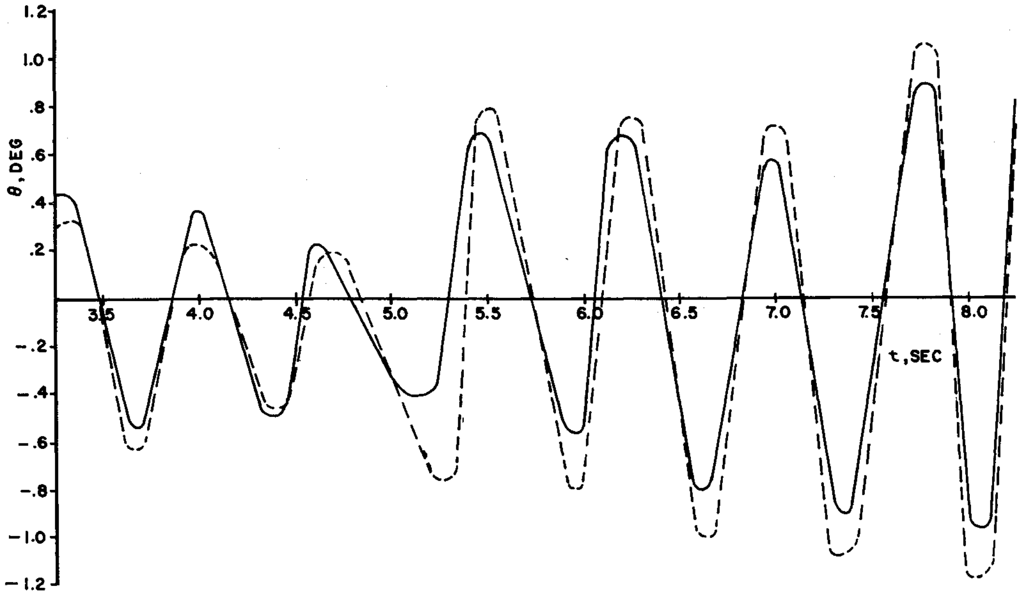


Fig. 5 Comparison of predicted pitch motion with experimental model data for head seas, V = 30 knots, prediction time T = 0.48 sec; — predicted, - - MIT experimental data.

and the Fourier transform of this kernel is

$$\bar{K}_\theta(\omega_e) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \bar{G}(\nu) \frac{d\nu}{\omega_e - \nu} \tag{42}$$

which is to be interpreted as a principal value integral. The truncated kernel function can be represented by

$$K_\theta(t) = G(t)u(t - T) \tag{43}$$

and its Fourier transform is given by

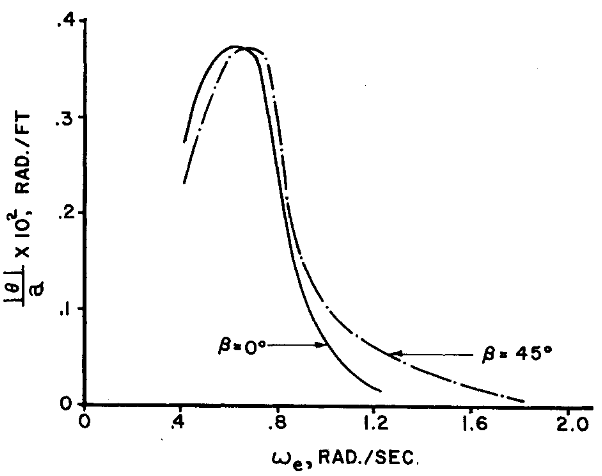
$$\bar{K}_\theta(\omega_e) = \frac{e^{-i\omega_e T}}{2\pi i} \int_{-\infty}^{\infty} G(\nu) \frac{e^{i\nu T}}{\omega_e - \nu} d\nu \tag{44}$$

Comparison of Eqs. (42) and (44) shows that, for small values of T, there is not much difference in the transforms of the kernel function for small values of the frequency, with the major differences arising for larger frequencies. Similarly it can be shown, by application of the Tauberian properties of Fourier transforms, that the values of the kernel function for small time are given by information obtained from its Fourier transform at large frequencies,[14,15] as exhibited by

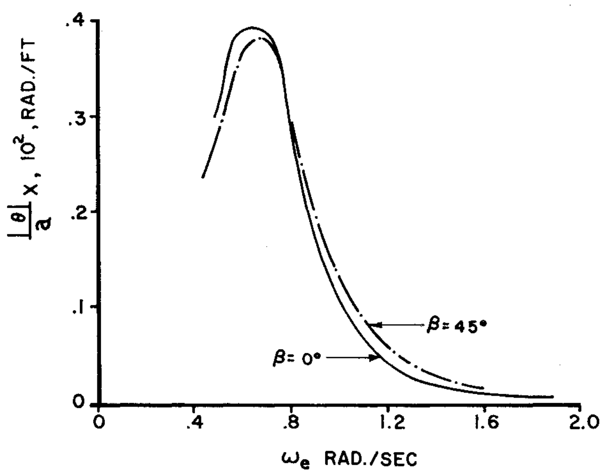
$$\lim_{t \to 0^+} K_\theta(t) = \lim_{\omega_e \to \infty} i\omega_e \bar{K}(\omega_e) \tag{45}$$

Since the relations of Eq. (40) define the Fourier transform of the kernel function to represent the pitch transfer function relative to wave amplitude, the conclusions of this analysis are that the truncation of the kernel function is equivalent to altering contributions of the higher-frequency portions of the transfer function. Since the aircraft carrier has very little response at higher frequencies, this neglect will have little practical effect, as is verified by the operations of physical realizability and truncation applied to the kernel functions obtained by transforming the frequency response functions.

One of the significant features of the ship transfer functions, in heave and pitch for an aircraft carrier of the Forrestal class, was the small differences due to the effect of heading between the ship and the waves, up to angles of 45° off the bow, when plotting the amplitude and phase characteristics as functions of the frequency of encounter ω_e.[1] Particular examples of this relation are shown in Fig. 6, which exhibits the pitch transfer function amplitudes for forward speeds of 20 and 30 knots, for headings of 0° and 45°. As a result of this characteristic it is expected that the kernel functions defined by Eqs. (31) and (32) for heading angles (β) up to 45° will have only a small dependence upon the heading angle. An illustration of this effect is shown in Fig. 7, which represents the kernel functions for headings that differ by 45°. The basic difference is a small phase shift, which can be expected in accordance with the basic definition of the function $F(\omega_e)$ defined in Eq. (4). Thus, it appears that this insensitivity to heading will allow adequate prediction of ship motion for unidirectional oblique waves, when measuring the wave input at a fixed point ahead of the bow. Since



a) V = 20 knots



b) V = 30 knots

Fig. 6 Pitch amplitude variation with encounter frequency and heading.

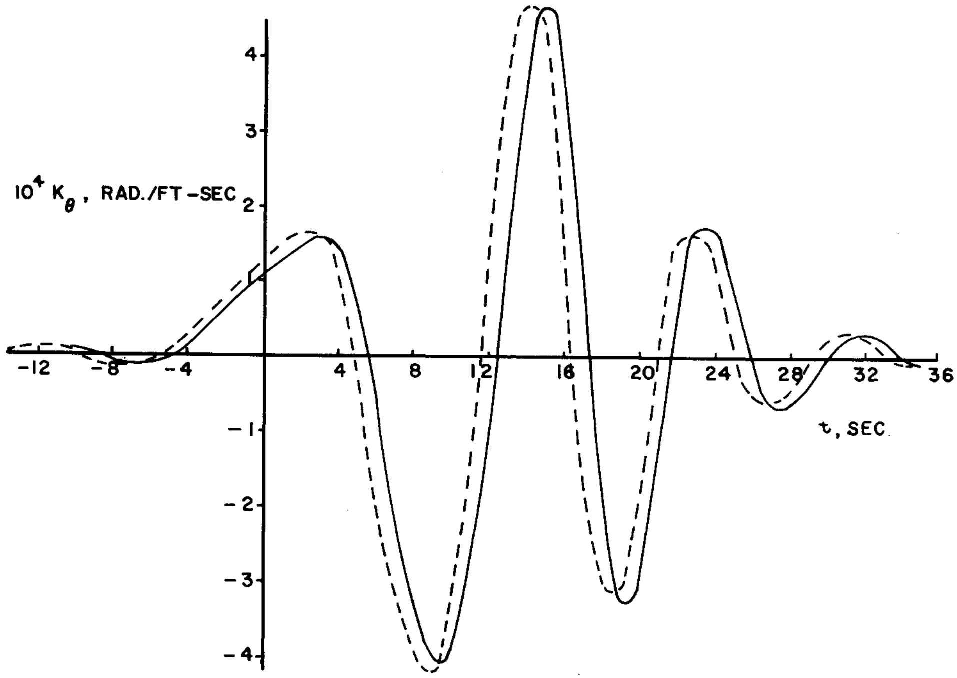


Fig. 7 Full-scale pitch kernel functions for different headings, $V = 20$ knots; — $\beta = 0°$, - - $\beta = 45°$.

complex short-crested seas are formed by a sum of various unidirectional irregular waves coming from various oblique headings, it can be expected that the prediction technique will function effectively in realistic short-crested seas. Since the carrier will always head into the wind during landing operations, most of the seas encountered will be bow seas for the predominant wave systems, and this technique will thereby be appropriate to realistic operational conditions.

The preceding statements are based upon the observed insensitivity of theoretical transfer functions to heading, but this must be verified by experiment before the technique can be expected to be useful in the manner described. To obtain some information on this, limited tests were carried out at Stevens Institute of Technology[16] in oblique waves, and an analysis of other available data[13] was made, which resulted in verification of the insensitivity to heading.

The input data for the prediction technique is the wave height at a point ahead of the ship bow, and as such it will (ideally) be distorted by waves that are generated by the ship's dynamic motions (pitch, heave, etc.) as well as reflections of the oncoming waves by the ship itself. The wave elements that can distort the input signal are those waves that are propagated forward of the ship, and according to the theory of oscillating translating sources[17] such waves will exist when the dimensionless parameter $\omega_e V/g < \frac{1}{4}$. In the case of regular waves the conditions for the occurrence of such waves can be determined precisely, but for an irregular pattern there may be some wave energy in a frequency band that could result in waves propagated forward. However, for the speed range of aircraft carriers, there is very little significant wave distortion due to the limited wave energy in the necessary frequency band (e.g., for $V = 20$ knots, ω_e values < 0.238 are necessary for the occurrence of forward propagated waves). This lack of significant wave interference is supported by experimental data in this program, where wave height data were simultaneously measured by a transducer located far to the side (at the same x-coordinate location) and limited comparisons made with the bow wave transducer data.

Statistical Prediction Theory

The Wiener prediction technique for stationary time series determines an optimum linear predictor on the basis of minimizing the mean-square prediction error. The final solution to the prediction problem developed by this method is a linear mathematical operation on the past history of the signal, which yields a short-time prediction in the future. The signal that is predicted is random, and is characterized only by statistical parameters. The most important characteristic required is knowledge of the power spectrum of the desired function.

Assuming a complete knowledge of the power spectrum of the signal, this function must be approximated as a rational function of frequency. The most crucial operation is the spectral factorization of the power spectrum into a product of two conjugate functions, and the selection of the portion of an integrand function that will have no poles in a certain region of the complex frequency plane. The transfer function of the optimum predictor is determined from this last result. The predictor transfer function (for prediction time T), denoted as $J_T(\omega_e)$, is complex function of frequency that must be compared to the ideal prediction transfer function $e^{i\omega_e T}$ so that the error

$$E_T = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi(\omega_e)|J_T(\omega_e) - e^{i\omega_e T}|^2\, d\omega_e \qquad (46)$$

will be a minimum, where $\Phi(\omega_e)$ is the power spectrum of the random variable to be predicted. This method results in an inherent error that increases as the prediction time is increased, and which depends only on the power spectrum of the input to the predictor. Other significant aspects of this prediction technique are the physical realizability property of $J_T(\omega_e)$, which is insured by the procedure, and the restriction that there is no extraneous noise in the recorded signal. Particular applications of this technique to aircraft carrier motions, wave records, and related low-frequency phenomena have been carried out,[18,19] and the results indicate a fair degree of pitch prediction for the order of 5 sec, with the error increasing significantly if the prediction time is increased much beyond this.

The main difficulty in implementing the Wiener prediction techniques is the necessity for complete knowledge of the power spectrum of the signal to be predicted. The determination of power spectra of low-frequency random processes is a complex operation, and requires processing of observed data for a time interval of approximately 20 min to obtain an accurate representation. The power spectrum must be represented as a rational function of frequency, and the processes of spectral factorization and other operations performed to arrive at the transfer function of the predictor. The re-

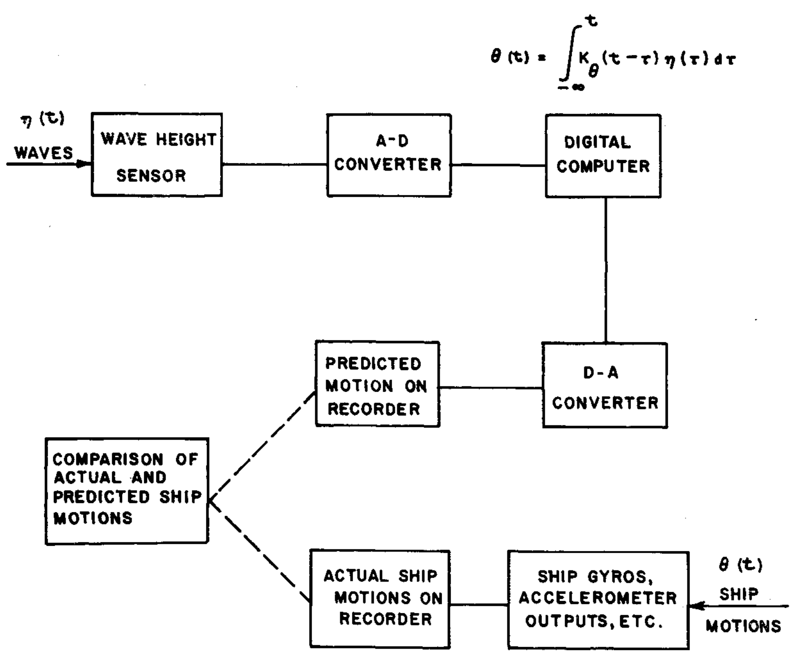


Fig. 8 Proposed signal processing and prediction system.

quired equipment for these operations is complicated and expensive, and the final implementation of the predictor circuit as an electrical network or analog computer will also be difficult to achieve, if these operations are to be carried out on a carrier for on-line prediction purposes. This last statement is true even if the power spectrum of motion does not change significantly for a few hours. However, there are significant changes in the power spectrum of pitch motion due to the effects of varying oceanographic conditions; changes in forward speed; heading variations; varying wind conditions; etc., under realistic operational conditions, and the implementation of a predictor is further complicated by these effects. In view of the foregoing, the use of Wiener prediction techniques has only limited application to aircraft carrier motion prediction.

Since the truncated kernel technique results in good prediction capability when operating upon the measured waves as an input, it appears possible to extend the prediction time of ship motion by obtaining a predicted wave input and operating upon this enlarged input data with the known kernel function predictor. Since the kernel function acts in such a way as to filter certain portions of the input data (i.e., the results appear to be independent of higher-frequency contributions), it is possible that prediction of the wave input can be made with sufficient accuracy. Then the composite operation of use of the kernel function operating on a wave input made up of the previous wave history, together with a predicted value for a few seconds ($\simeq$3 sec ahead), can result in an enlarged prediction time of the order of 9 sec. The basis for expecting a simple treatment of prediction of the wave input is the fact that the most significant parameter required for adequate prediction by the Wiener technique is a knowledge of the central frequency,[19] i.e., the frequency for the largest value of spectral energy. It appears easier to determine this particular quantity, and the approach is further enhanced by the limited sensitivity to other parameters such as bandwidth, high-frequency cutoff rate, etc., because of the ultimate filtering action of the kernel function. Further investigation of this possibility will be carried out in the near future to determine how effective a prediction of the waves, and ultimately the ship motion, can be obtained.

Application of Prediction Theory

In view of the success in predicting carrier pitch motions up to 6 sec ahead, as well as the possibilities inherent in other prediction techniques applied to the waves, this technique should be extended to operational evaluation in full scale. It is necessary to provide a computer for real-time processing of the truncated kernel predictor method, and also to have a sensor system that provides the wave time history at the bow as the basic input. Various wave height sensors developed for the Navy Department can provide the latter. The particular wave height sensors use radar and ultrasonic waves to determine the surface wave time history,[20] with compensation networks to correct for the motion of the transducer itself, since it is located at the bow of a ship. These systems have the required accuracy for determining a continuous wave time history as input data, which can then be used with known kernel functions (stored in a digital computer memory) to carry out the required mathematical operations. A limited storage of kernel function data is necessary (for approximately 36 sec of time) and the necessary computations can be carried out by a small general purpose digital computer. The availability of the wave measuring device, as well as recent developments in high-speed computation, are important aspects of this prediction technique.

With regard to prediction of the wave motion input, an alternate method may be used that is more fruitful both conceptually and for direct application using the digital technique. The wave input prediction method is an application of the techniques of recursive filtering and prediction applied to a sequentially sampled digital input.[21] This new technique, using the Kalman filter, is applicable to both continuous data and sampled data, and the proposed digital format with sampled inputs will be appropriate for this problem when the necessary hybrid elements (A-D and D-A converters) are included. The Kalman filter approach is presently being investigated in order to determine the least number of parameters for successful prediction of the wave time history. Following this development, a simple composite operation based on wave input data into a computer that contains a preprogramed kernel function, as well as other simple operations that may require a selective input (such as forward speed, or observed central frequency, etc.) will be the final technique for the over-all procedure. A full-scale evaluation of this prediction technique is planned for the 1968–1969 winter period, with the USS Independence as the test vessel. An illustration of the data acquisition and signal processing system that will be used to evaluate the performance of this technique is shown in Fig. 8. In view of the success shown by the preliminary results described in this paper, this program offers promise of useful prediction capability when implemented in a full-scale installation under operational conditions.

Conclusions

The major result of this study is the development of a means of calculating ship motion time histories, including predicted values for a short time ahead, by means of a convolution integral representation. A kernel function, derived from ship response functions, is modified by certain truncation operations and is then applied to operate on input data in the form of the present and past history of wave motion measured at the bow of the ship. The significant features of the kernel function technique, in contrast with a statistical technique such as the Wiener method, are shown to be an independence of the statistical characteristics of the waves and the ship motion itself; small dependence upon heading angle in the practical operational regime; ease of implementation using a high-speed digital computer; and a limited dependence on forward speed.

The direct application of this deterministic prediction method yields a prediction time of about 6 sec, and means of increasing this time are considered. An attractive approach is the use of modern prediction techniques, such as the Kalman filter method, applied to the wave motion input so that the kernel function will tend to smooth some of the prediction errors inherent in that approach. In view of the proposed digital format of the prediction method, this technique can be readily incorporated into the over-all procedure. Thus, the prediction time can possibly be extended up to about 9 sec by this method. A full-scale evaluation at sea of this over-all technique will be carried out in early 1969.

The predicted ship motion data, when obtained as part of an operational system on board an aircraft carrier at sea, can be incorporated into the landing operation. In application to manually controlled landings, it will serve as a valuable aid to the Landing Signal Officer (LSO) for earlier and more precise determination of waveoff criteria associated with landing on a moving deck. The information will similarly be useful in the Automated Carrier Landing System by providing more advanced information on terminal conditions and thereby contributing to the reduction of landing accidents.

It is anticipated that successful performance of the prediction system described in this report will increase the safety of the landing operation and will also extend the range of sea conditions in which carrier operations will be possible. A system such as that proposed herein has a greater possibility of success, and appears easier to implement, than other suggested means for controlling the ship motion environment such as addition of antipitching fins, which only have limited motion reduction capabilities; introduce additional disturbances; and/or require fitting the ship with large appendages, additional powering units, etc. Thus, the practical implementation means of the proposed prediction system is another factor in its favor as a method for mitigating the influence of the sea environment on the landing operation.

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